Exact critical exponents for the antiferromagnetic quantum critical metal in two dimensions

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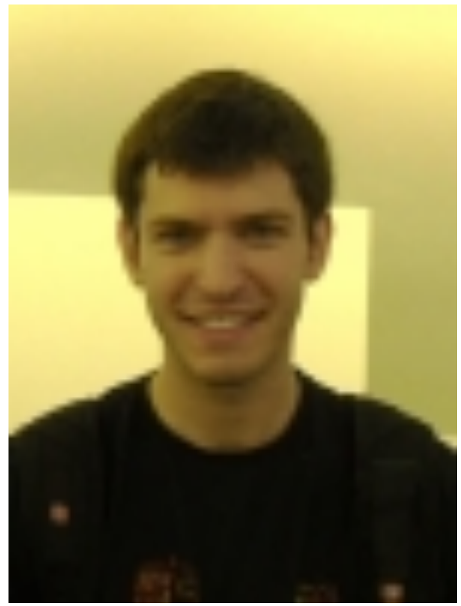
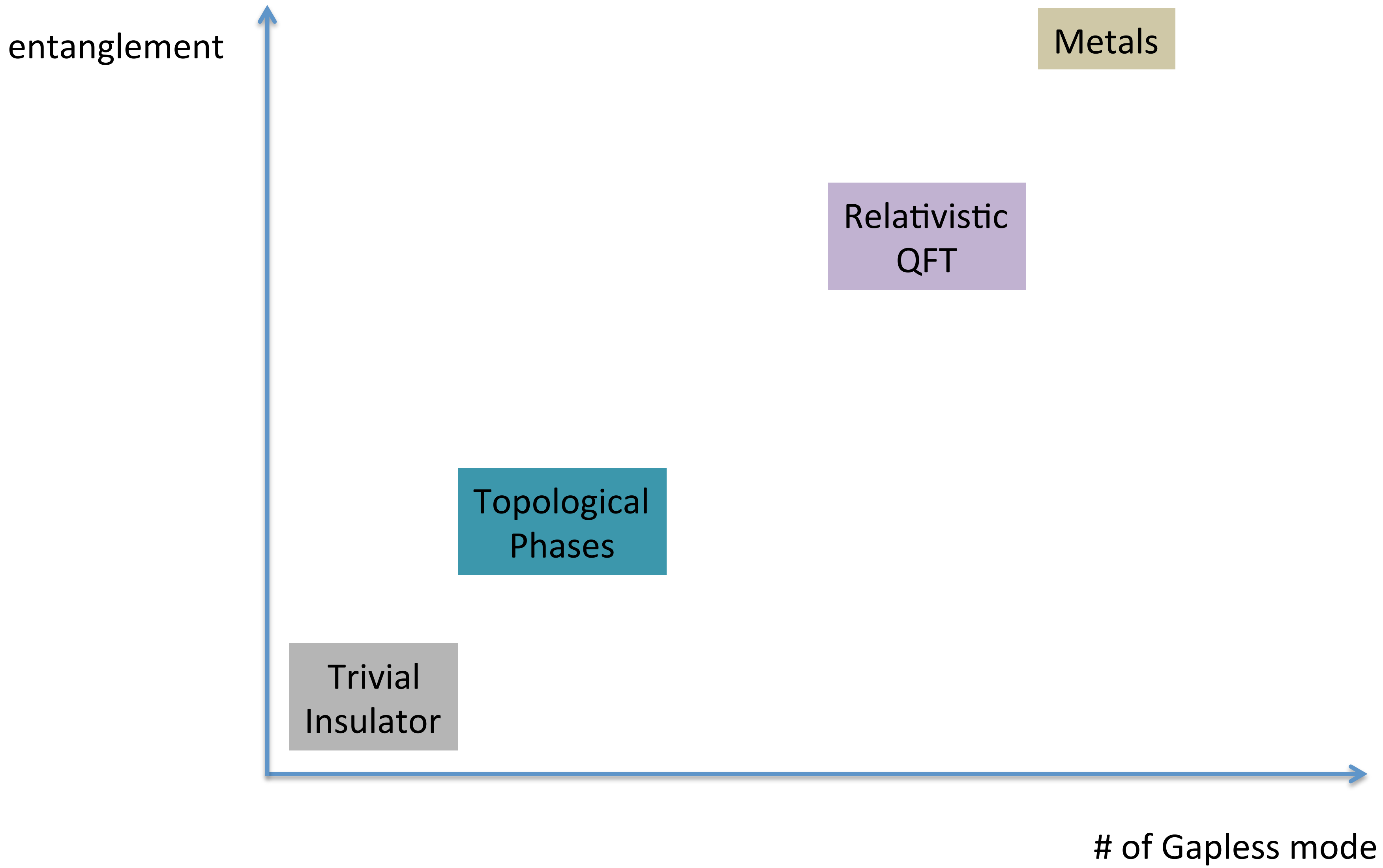


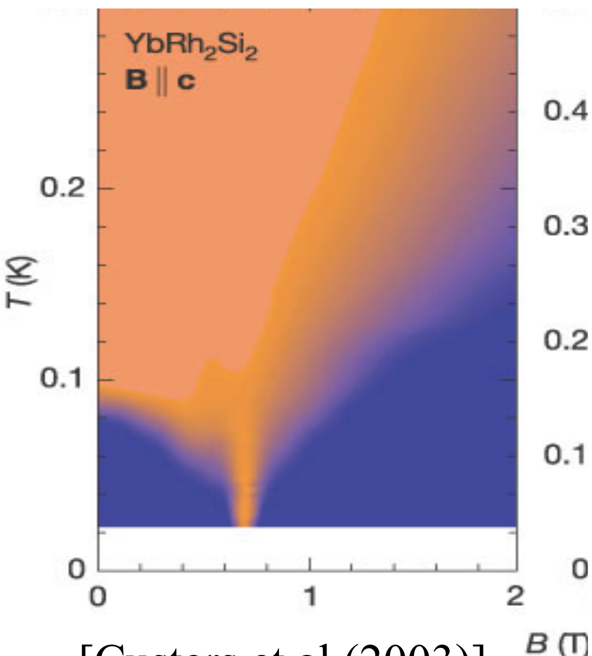
Peter Lunts

Andres Schlief

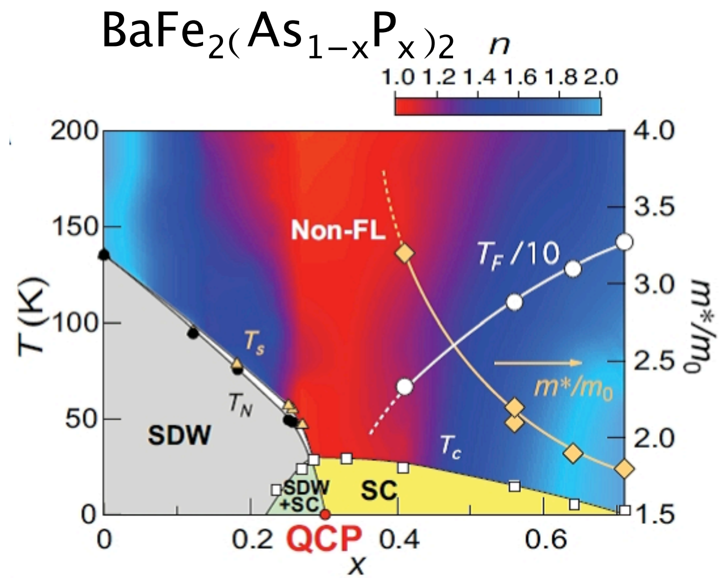
Quantum matter



Breakdown of Fermi liquid near Quantum Critical Point



[Custers et al.(2003)]

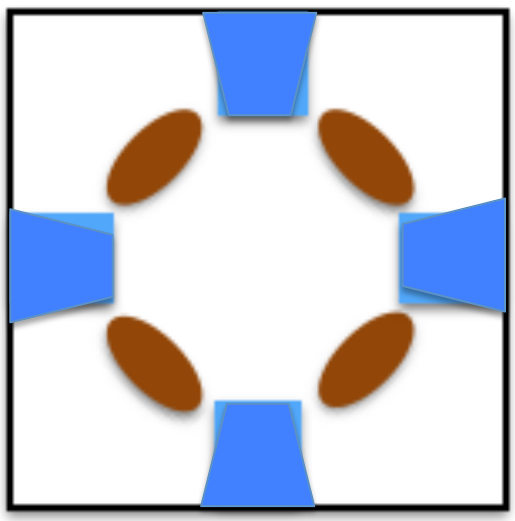


[Hashimoto et al. (2012)]

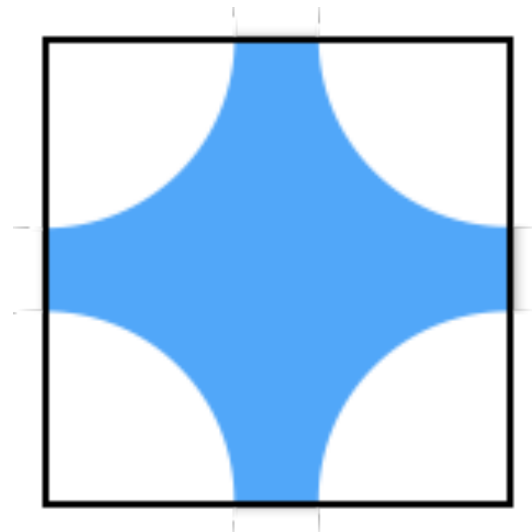
Goal : capture universal low-energy physics of metals
without well-defined quasiparticle

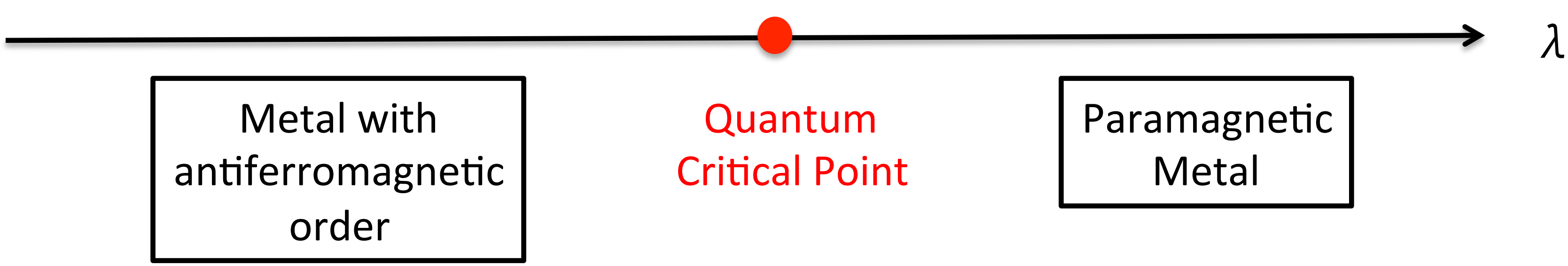
Antiferromagnetic phase transition in metal

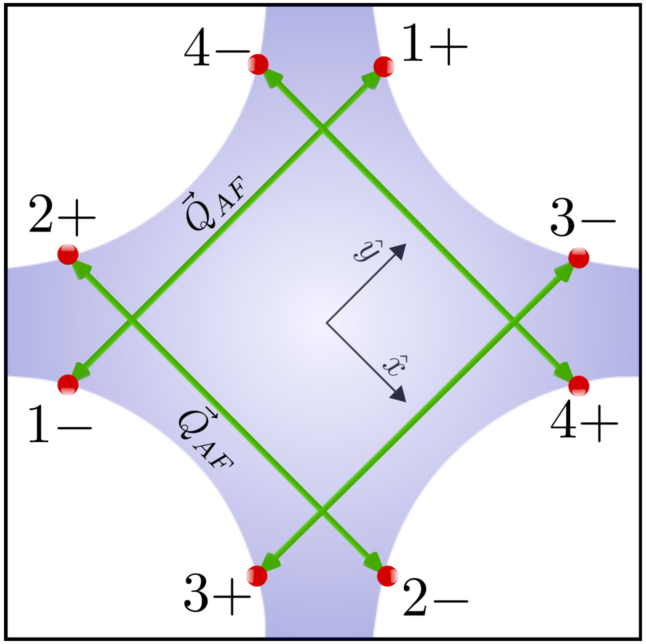
$$\vec{\phi} \neq 0$$



$$\vec{\phi} = 0$$







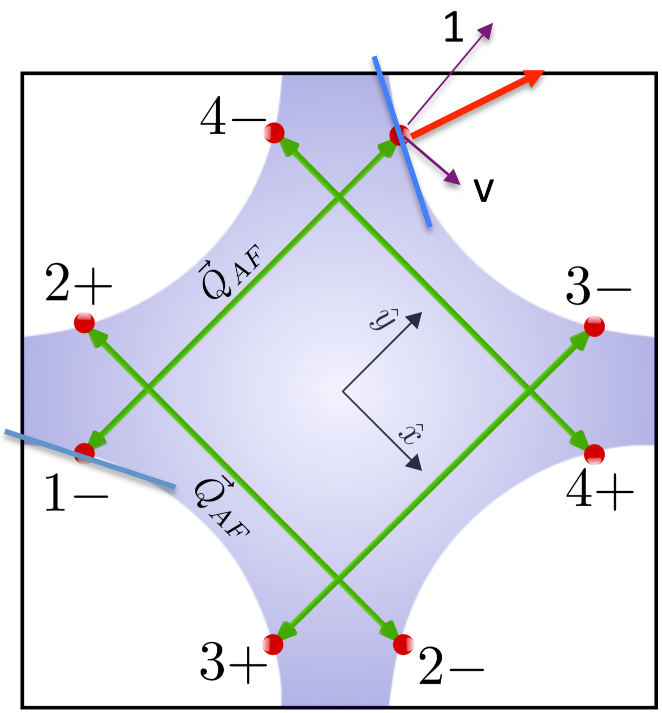
Minimal Theory

$$e_1^{\pm}(\vec{k}) = -e_3^{\pm}(\vec{k}) = vk_x \pm k_y$$

$$e_2^{\pm}(\vec{k}) = -e_4^{\pm}(\vec{k}) = \mp k_x + vk_y$$

$$\begin{aligned}\mathcal{S} \; = & \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \; \psi_{l,\sigma}^{(m)*}(k) \left[ik_0 \; + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k) \\ & + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2 |\vec{q}|^2 \right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q) \\ & + g_0 \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \; \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c. \right]\end{aligned}$$

Parameters of the theory



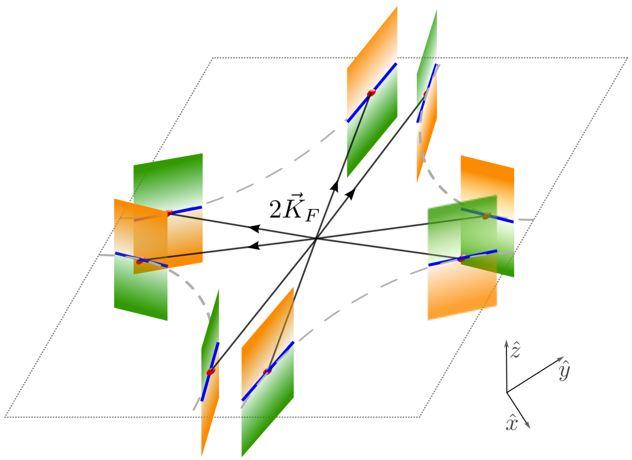
- v : Fermi velocity perpendicular to Q_{AF}
- c : boson velocity
- g : coupling bet'n fermion and boson

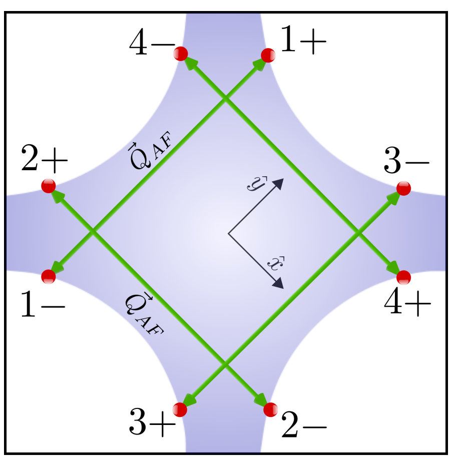
- If v=0, hot spots connected by Q_{AF} are nested

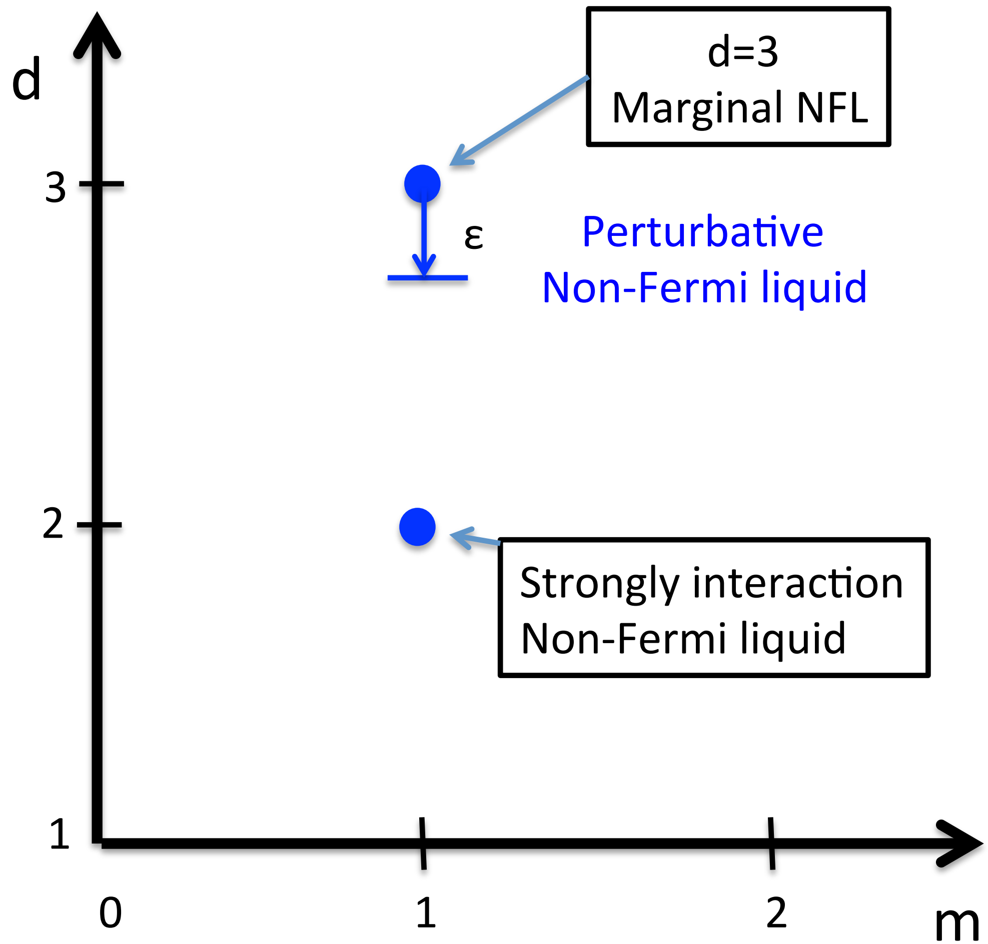
Earlier works (incomplete list)

- Fermi surfaces get nested and quasiparticle is destroyed near the hot spots [Abanov, Chubukov,Schmalian]
- The theory flows to strong coupling regime even in the large N limit [Metlitski, Sachdev]
- The field theory can be regularized by a sign-problem-free lattice model : QMC shows enhancement of d-wave SC at QCP [Berg,Metlitski, Sachdev; …]
- The precise nature of the NFL state has not been understood due to a lack of control over the theory

A continuous interpolation between 2d Fermi surface and 3d metal with line nodes







[Sur, Lee (14)]

Lesson from the ε-expansion

[Sur, Lee (14); Lunts, Andres, Lee(17)]

- ε-expansion ≠ loop expansion
- Emergent quasi-locality with a hierarchy in velocities

$$v, c \to 0 \ \left(\frac{v}{c} \to 0\right), \quad g \to 0 \left(\frac{g^2}{v} \to O(\epsilon)\right)$$

- Collective mode is strongly damped by particle-hole excitation and acquire an O(ε) anomalous dimension
- Fermions remain largely coherent

Ansatz in 2+1D : Interaction driven scaling

Scaling which leaves the interaction marginal at the expense of dropping kinetic energy as irrelevant term

$$\mathcal{S} = \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \, \psi_{l,\sigma}^{(m)*}(k) \left[ik_0 \; + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k)$$

$$+\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2 |\vec{q}|^2 \right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q)$$

$$+g_0 \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c. \right]$$

Ansatz in 2+1D : Interaction driven scaling

Drop the boson kinetic term because collective mode is damped by the particle-hole excitations

$$\mathcal{S} = \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \, \psi_{l,\sigma}^{(m)*}(k) \left[ik_0 + e_l^m(\vec{k})\right] \psi_{l,\sigma}^{(m)}(k)$$

$$+\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2|\vec{q}|^2\right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q)$$

$$+g_0 \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c.\right]$$

Ansatz in 2+1D : Interaction-driven scaling

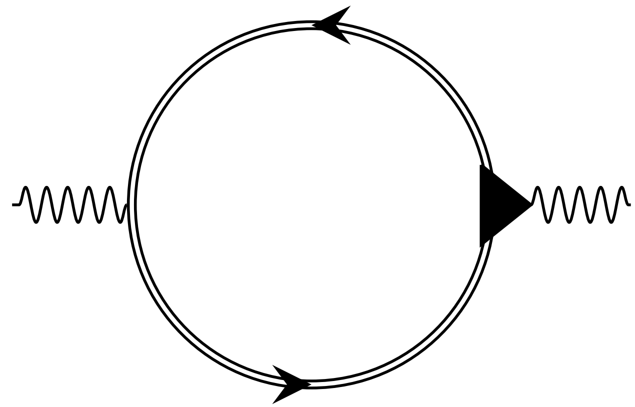
$$[k_0] = [k_x] = [k_y] = 1,$$

$$[\psi(k)] = [\phi(k)] = -2.$$

- Electron keeps the classical scaling dimension
- Collective mode has a large anomalous dimension
- It turns out that these are exact

[Andres, Lunts, Lee (17)]

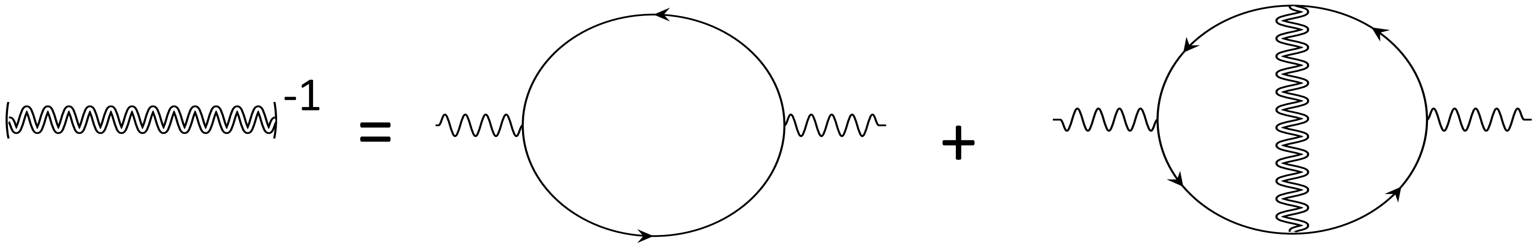
Self-consistent boson propagator



$$D(q)^{-1} \;=\; m_{CT} - \pi v \sum_n \int dk \;\mathrm{Tr}\left[\gamma_1 G_{\bar{n}}(k+q)\Gamma(k,q)G_n(k)\right]$$

- In general, it is hard to solve the self-consistent equation because G(k), Γ(k,q) depend on D(q)

Small v limit



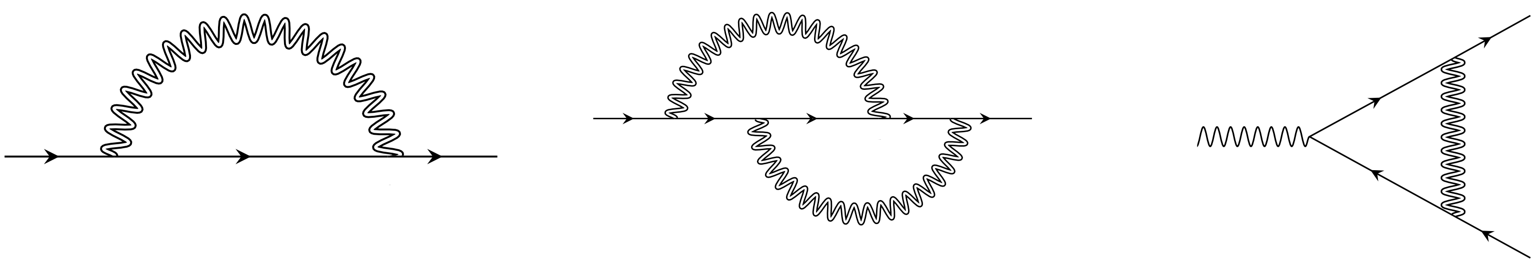
$$D(q)^{-1} = |q_0| + c(v)\Big[|q_x| + |q_y|\Big],$$

$$c(v) = \frac{1}{4}\sqrt{v\log(1/v)}$$

- Boson propagator is entirely generated from particle-hole fluctuations
- v << c << 1 in the small v limit

Flow of v

- In the small v limit, v indeed flows to zero in the low energy limit, which completes the cycle of self-consistency



$$\frac{dv}{d\ln\mu} = \frac{6}{\pi^2}\, v^2 \log\left(\frac{1}{c(v)}\right) \qquad v = \frac{2\pi^2}{3}\left(\log\frac{1}{\mu}\ \log\log\frac{1}{\mu}\right)^{-1}$$

Dynamical spin susceptibility

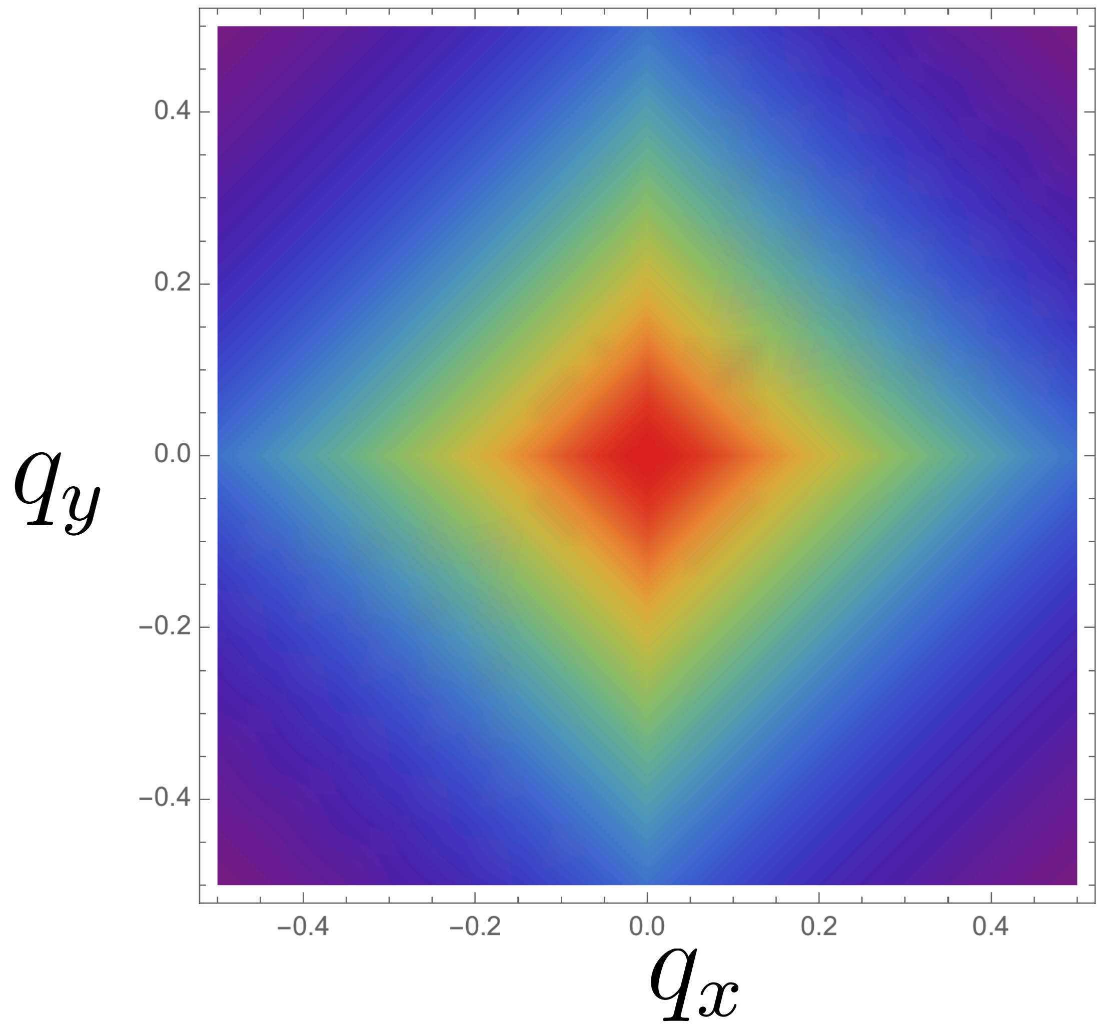
$$D^R(\omega, \vec{q}) = \frac{1}{F_\phi(\omega)\left(-i\omega F_z(\omega) + \frac{\pi}{4\sqrt{3}}\frac{|q_x|+|q_y|}{\left(\log\frac{1}{\omega}\right)^{1/2}}\right)}$$

$$F_\phi(\omega) = e^{\frac{2}{\sqrt{3}}\left(\log\frac{1}{\omega}\right)^{1/2}}$$

$$F_z(\omega) = e^{2\sqrt{3}\frac{\left(\log\frac{1}{\omega}\right)^{1/2}}{\log\log\frac{1}{\omega}}}$$

- Spin fluctuations are strongly damped by particle-hole excitations

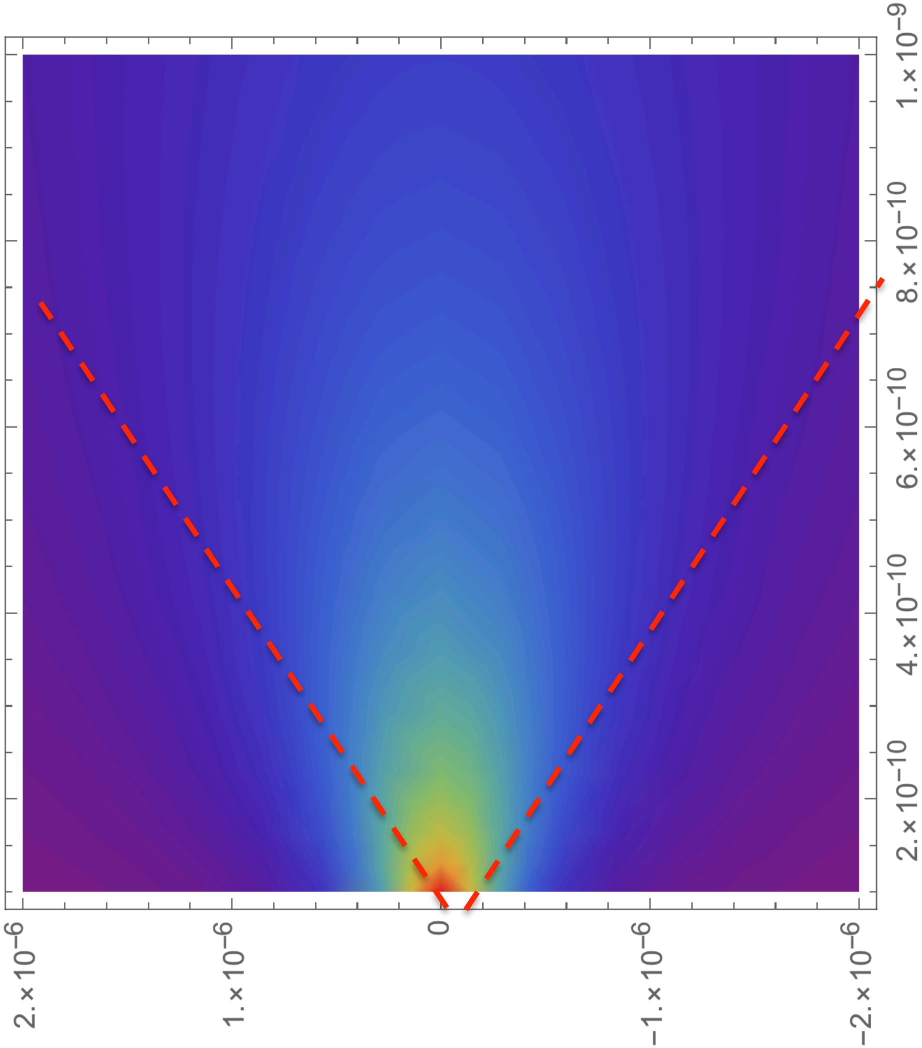
Dynamical Spin Susceptibility $\chi''(0^+, \vec{Q}_{AF} + \vec{q})$



- Only C_4 symmetric; no emergent O(2)

Dynamical Spin Susceptibility $\chi''(\omega, \vec{Q}_{AF} + \vec{q})$

ω



$$\omega \sim \sqrt{v}\, \Delta q$$

$|q_x| + |q_y|$

- Incoherent peak centered at Q_{AF} at all ω
- The width in momentum space scales linearly in energy

Spectral function at the hot spots

$$A(\omega) \sim \frac{1}{\omega e^{2\sqrt{3}\frac{\left(\log\frac{1}{\omega}\right)^{1/2}}{\log\log\frac{1}{\omega}}}}$$

- Weak deviation from Fermi liquid

Divergent correlation length

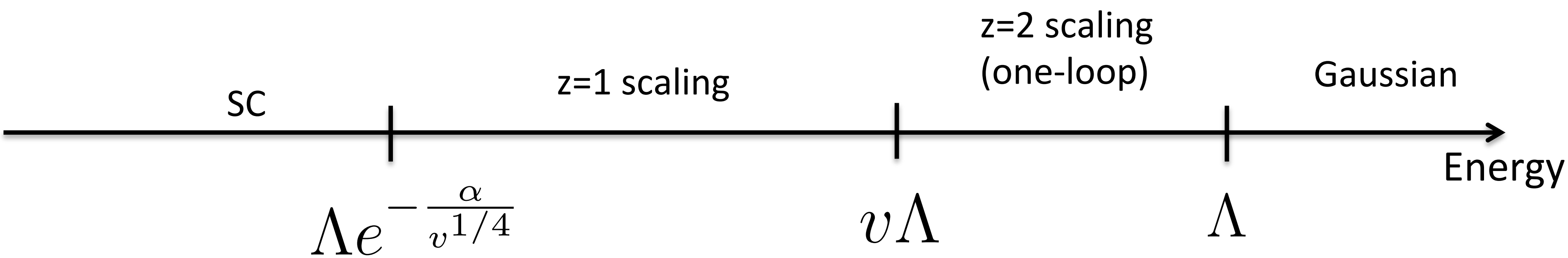
$$\xi \sim (\lambda - \lambda_c)^{-1} e^{\frac{2}{\sqrt{3}}\left(\log \frac{1}{|\lambda - \lambda_c|}\right)^{1/2}}$$

Specific heat

$$c \sim T e^{2\sqrt{3}\frac{\left(\log \frac{1}{T}\right)^{1/2}}{\log \log \frac{1}{T}}}$$

Superconductivity

- In the low T limit, d-wave superconductivity is enhanced
- Hierarchy in energy scales



- The crossover energy scales are sensitive to the bare value of v, which can be experimentally tested
- In the small v limit, there is a large window for the z=1 critical scaling

Summary

- Low Antiferromagnetic critical metal in 2+1D
  - Exact critical exponents are predicted based on a non-perturbative solution
  - Precise measurements are needed to test the predictions
- Open problems
  - Full applicability of interaction-driven scaling?
  - Transport, real time dynamics ?